Introductory Physics: Drawing inspiration from the mathematically possible to characterize the observable

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- Trevor Smith, Rowan University
- Philip Eaton, Stockton University
- Andrew Boudreaux, WWU
- Steven Kanim, NMSU

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- Paula Heron
- Peter Shaffer
- Donna Messina
- Anne Alesandrini
- Jesse Ashworth-Marin
- Jared Canright
- John Goldak
- Taylor Gurreithun
- Qiriu Guo
- Ella Henry
- Kristin Kellar
- Al Snow
- Charlotte Zimmerman

Other Plenary Speakers

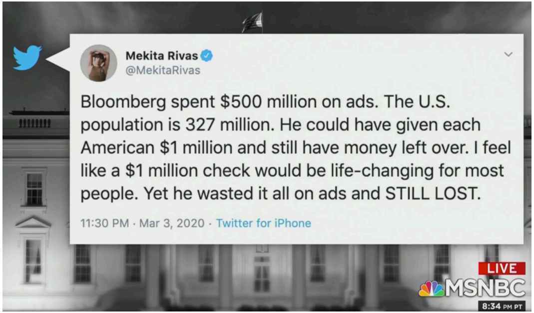
- Carrie, our assumptions about our students can do damage, discussion about math modeling cycles, and the framework of *saber* and *conocer* (savoir and connaître)
- Ranier, money and goods are fluids, business students are curious too, and benefit from conceptualizing calculus
- Marcy, symbolic forms, quantities in physical science are rooted in measurable quantities, generalized time
- Bryan, math+engineering=physics, so recall all the examples shared, and the techniques used for “collecting seashells"

Transformative Worldview

Quantitative literacy releases predictive powers.
Predicative powers are transformative.

Quantitative Literacy

Brian Williams (extremely well-known TV news presenter) in discussion with Mara Gay, a member of the editorial board of the New York Times:



Williams: When I read it tonight on social media, it kind of all became clear. Bloomberg spent $500 million on ads. U.S. Population, 327 million. Don't tell us if you're ahead of us on the math. . . he could have given each American $1 million and have had lunch money left over. It's an incredible way of putting it

Quantitative Literacy

The discussion went on for nearly a minute on national television, before they moved on to other news.

Someone on the show prepared the slide for him, and no one on the show stepped in to shut them down.

The eventual retraction:

> *Mara and I got the same grades at math...* ***I didn't have it in high school. I don't have it tonight.*** *I stand corrected. Sorry about that.*

Quantitative Literacy

- Global issues increasingly require quantitative reasoning to understand basic information.
- Medical decisions are quantitative. Medical schools have increased emphasis on "calculator-less" quantitative reasoning in their admissions process.
- Engineers and scientists routinely use quantities and their rates of change to make sense of the physical world, ***and it is a learned skill.***

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Quantitative sensemaking is not a standard outcome of a college education.

Calculus and adjacent disciplines can help

- A significant majority of calculus students will also take physics (in the US $\approx 75\%$)
- Main exception are students of economics and business.
- Arguably, all non-major calculus students will use quantities and their rates of change to make sense of the world, ***and it is a learned skill.***

Focus of conference

- How do biologists, chemists, economists, engineers and physicists understand and use calculus concepts in their disciplines?
- What does that imply for the teaching and learning calculus?

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Calculus in physics - focus on modeling

How do physicists understand and use calculus concepts in their disciplines?

Math/Physics Conceptual Blend

CALCULUS I

MATHEMATICS

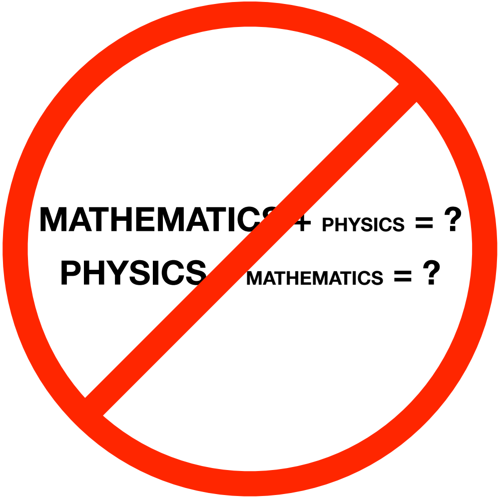
+ PHYSICS = ?

INTRO PHYSICS

PHYSICS

+ MATHEMATICS = ?

Math/Physics Conceptual Blend



Conceptual Blend

MATHEMATICS
PHYSICS

↓

MATHEMATICS
PHYSICS

↓

MA *P* TH *H* EM *Y* AT *S* ICS

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MAPTHHEMYATSICS

Conceptual Blend *(Fauconnier & Turner, 2008)*

MATHEMATICS
PHYSICS

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Conceptual Blend

MAPTHHEMYATSICS

Functions as storytelling

To an expert, a physics equation "tells the story" of an interaction or process.

$$x(t) = 40\text{ m} + (-5\text{ m/s})t + \frac{1}{2}(-9.8\text{ m/s}^2)t^2$$

- Experts might quickly construct a mental story of the co-variation of position and time of a projectile that starts 40 m above the ground and is launched with a speed of 5 m/s vertically downward.
- Part of the challenge of learning physics is developing the ability to decode symbolic representations in this manner.
- Facility with vector quantities and rate reasoning is essential **and challenging**.

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Beyond Introductory Level: student difficulties

There is mounting evidence that students struggle with conceptualizing arithmetic and algebra as used in introductory physics. These difficulties carry over into subsequent course taking.

Summary of studies on upper-division physics:

- Activating appropriate mathematical tool without prompting (e.g. delta function, Taylor series)
- Recognizing meaning of mathematical expressions
- Spontaneous reflection on results (e.g., limiting cases, dimensional analysis)
- Generating mathematical expressions from physical description

(Caballero, Wilcox, Doughty, and Pollock, 2015)

Beyond Introductory Level: student aspirations

The students had no problems with executing the mathematics when asked, but they expressed a strong desire to understand what they were doing, and why.

(Caballero, Wilcox, Doughty, and Pollock, 2015)

Beyond Introductory Level: do no harm?

I found it interesting that even as students advance through higher level physics classes, they don't really gain a better understanding . . . ***I honestly found it really reassuring.*** *As someone already in the thick of the 300 level courses, I never felt like I was actually improving, just learning more.* ***Its kind of a debilitation[sic] feeling.***

- junior physics major, self-identifies black and female

(Guo, Zimmerman, White Brahmia, under review)

Math world, physics world

Students perceive that "doing" for a math class and using math in physics are different

- "When you think about just, like, the pure math problems, that's all you really think about–just the fact that dx is just telling you . . . what variable to use . . . but . . . here, it represents, it represents something . . . " *(Bajracharya, Sealey and Thompson 2023)*
- "I have math and physics on different days so I forget about math when I go to physics, I forget about physics when I go to math." *(Taylor & Loverude, 2023)*

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Math world is part of the real world

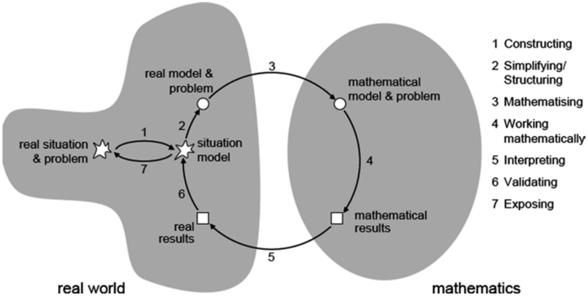


Figure: Separated world model
Blum and Leiß(2007)

- Expert physics modeling is a tight blend of physics and mathematical worlds *(Zimmerman et al. under review, Van den Eynde, 2021)*
- Mathematics education researchers observe a continuous contextual validation of mathematization, in contrast to the separate "world" model *(Czocher, 2016, Borromeo Ferri, 2007, Ärlebäck, 2009)*

Engineering students in Differential Equations

In a recent studies of engineering students solving problems

- Majority not confident in linking mathematics they were doing to context it represents *(Rowland, 2006)*
- Few students recognized the the units of each term in the model had to be the same *(Rowland, 2006)*

Interpreting *(Czocher, 2016)*
Re-contextualizing the result

- Referring to units
- Interpreting meaning from from equation elements

Validating*(Czocher, 2016)*
Verifying result against constraints

- Extreme/special cases
- Model limitations
- Dimensional analysis

Going Around Gainesville, *Hobson and Moore, 2017*

Some Georgia students have decided to road trip to Tampa Bay for spring break, which means traveling around Gainesville on their way down and back. Create a graph that relates the total distance traveled and their distance from Gainesville during the trip.

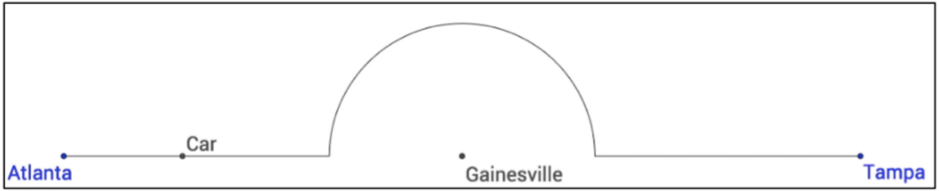
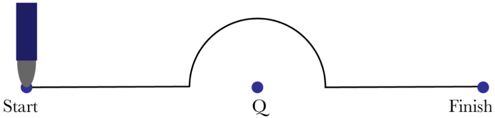


Figure 1. The *Going Around Gainesville* (*GAG*) task.[1]

Expert Physics Graphical Modeling

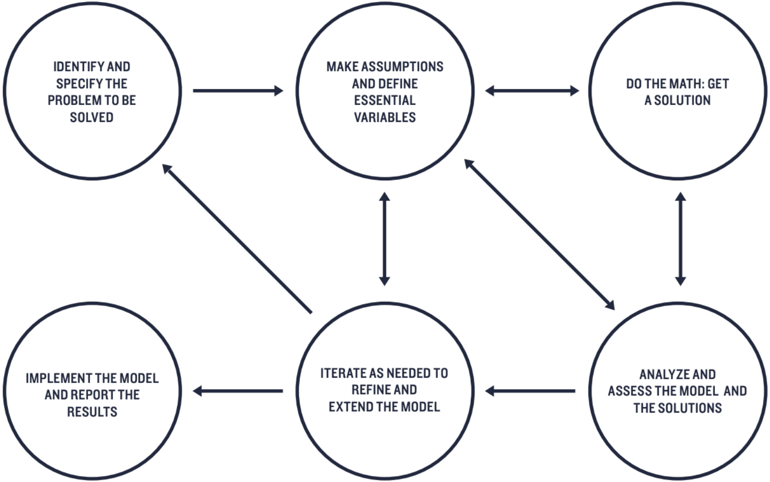
(Zimmerman, Olsho, Loverude and White Brahmia, under review)

- Replication/extension of study of mathematics expert covariation with graphing tasks (Hobson & Moore, 2017)
- Physics experts saw same items, and isomorphic items in less tangible physics contexts
- The mental actions students attempted in Czocher study, coupled to a continuous attention to quantity, were seen as essential devices that led to success for experts.



More Accurate Model of Modeling

From *GAIMME, Society for Industrial and Applied Mathematics, (Society of Industrial and Applied Mathematics)*



Quantities in physics

- Students encounter over 100 new quantities in an introductory physics course, and immediately encounter them in symbolic models
- Experts blend mathematical concepts and physical quantities unconsciously and seamlessly *(Zimmerman et al, under review, Kustuch, Roundy, Dray, & Manogue, 2014)*
- Physics instruction assumes students are familiar with quantification and conceptualizing **multiplicative structures** (arithmetic operations, ratios, proportions, linear functions, dimensional analysis and vector spaces; *Vergnaud, 1998*)

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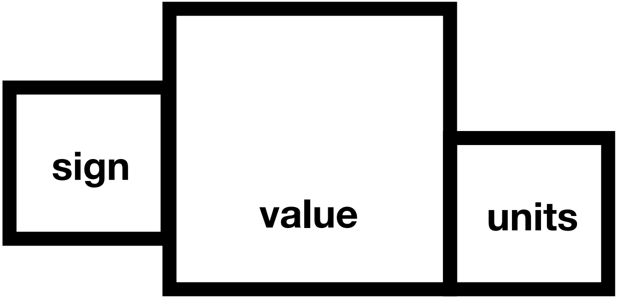
Physics quantities carry mathematical meaning

Quantities involve a value, units and usually a sign–which all carry physical significance. Quantity symbolic form *(White Brahmia 2019))*



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A motion is characterized by - 9.8 m/s^2*. What does the value 9.8 represent? What does the sign mean?*

Physics quantities carry mathematical meaning

A motion is characterized by -9.8 (m/s)/s What does the value 9.8 represent?What does the sign mean?

Physics quantities carry mathematical meaning

A motion is characterized by -9.8 m/s^2. What does the value 9.8 represent?What does the sign mean?

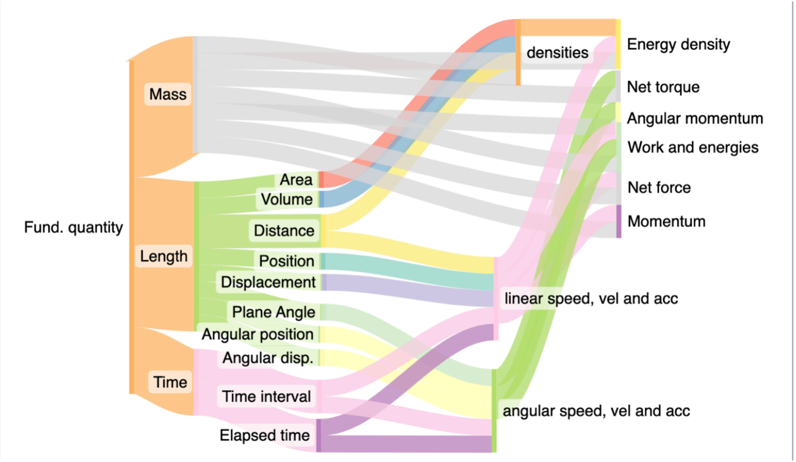
A motion is characterized by 0.1 s^2/m. What does the value 0.1 represent?

Physics quantities carry mathematical meaning

A motion is characterized by -9.8 m/s^2. What does the value 9.8 represent?What does the sign mean?

A motion is characterized by 0.1 s/(m/s). What does the value 0.1 represent?

Quantities in physics



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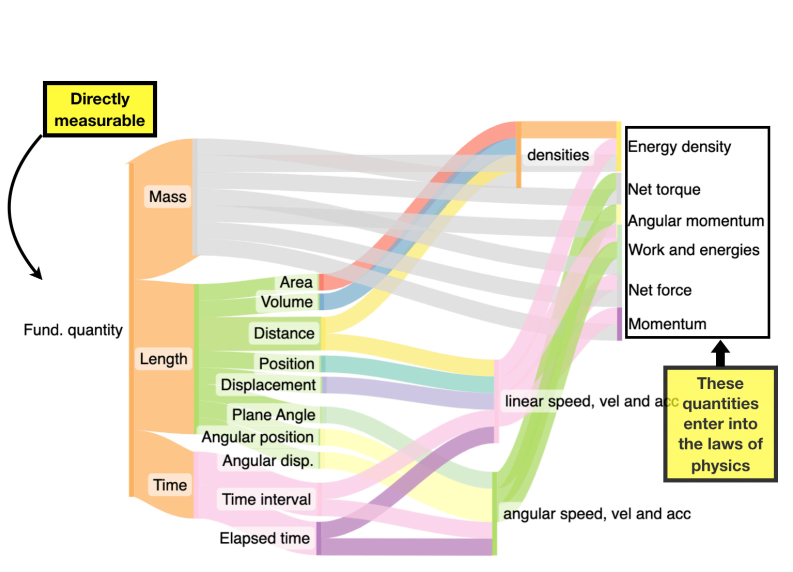

Quantities in physics

Quantities in physics:

- are derived from seven fundamental quantities, which are directly measurable. The rest are generated through mathematical operations.
- commonly have an amount/change/rate/accumulation relationship.

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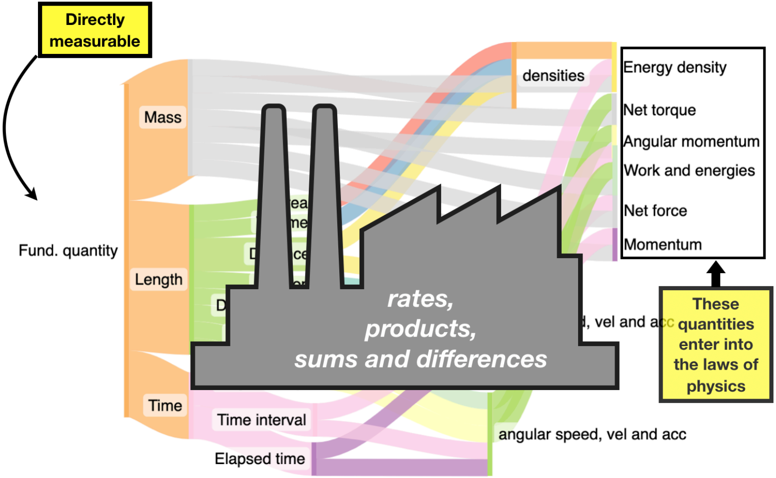

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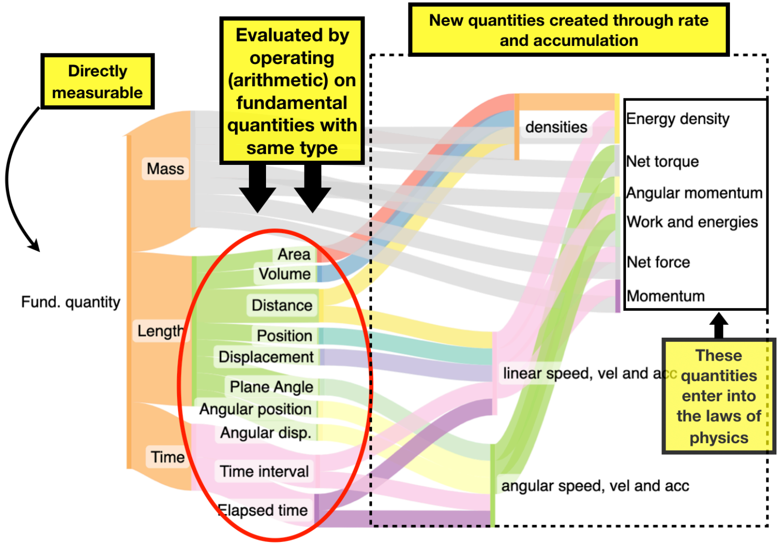

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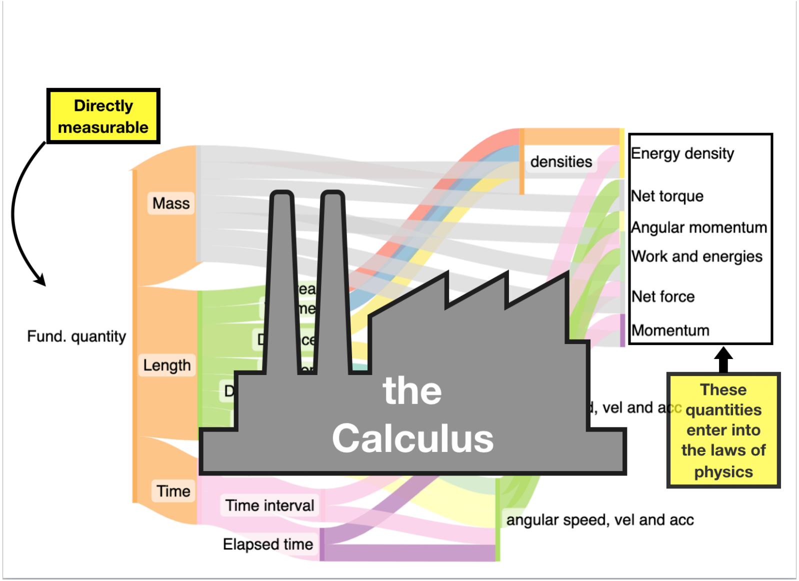

Calculus in physics - focus on modeling

What does that imply for the teaching and learning calculus?

ACRA framework, *(Samuels, 2022)*

ACRA = Amount Change Rate Accumulation

Fundamental Theorem of Calculus (FTC): $f(b) - f(a) = \int_a^b f'(x)\,dx$

Quantity	$f(b) - f(a)$	$= \int_a^b df$	$= \int_a^b \frac{df}{dx}\,dx$	$= \int_a^b f'(x)\,dx$
	total change (accumulation)	infinite sum of every infinitesimal change	integral of every (infinitesimal change/ infinit. input change) *infint. input change	the integral of infinitesimal rate times infinitesimal input change

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Physics Quantities and the FTC

Quantity	$f(b)-f(a)$	$=\int_a^b df$	$=\int_a^b \frac{df}{dx}\,dx$	$=\int_a^b f'(x)\,dx$
displace-ment	$x(t_2)-x(t_1)$	$=\int_{t_1}^{t_2} dx$	$=\int_{t_1}^{t_2} \frac{dx}{dt}\,dt$	$=\int_{t_1}^{t_2} v(t)\,dt$
$\Delta\mathbf{v}$	$v(t_2)-v(t_1)$	$=\int_{t_1}^{t_2} dv$	$=\int_{t_1}^{t_2} \frac{dv}{dt}\,dt$	$=\int_{t_1}^{t_2} a(t)\,dt$
impulse	$p(t_2)-p(t_1)$	$=\int_{t_1}^{t_2} dp$	$=\int_{t_1}^{t_2} \frac{dp}{dt}\,dt$	$=\int_{t_1}^{t_2} F(t)\,dt$
work done *on* system	$U(x_2)-U(x_1)$	$=\int_{x_1}^{x_2} dU$	$=\int_{x_1}^{x_2} \frac{dU}{dx}\,dx$	$=\int_{x_1}^{x_2} F(x)\,dx$

(White Brahmia 2023)

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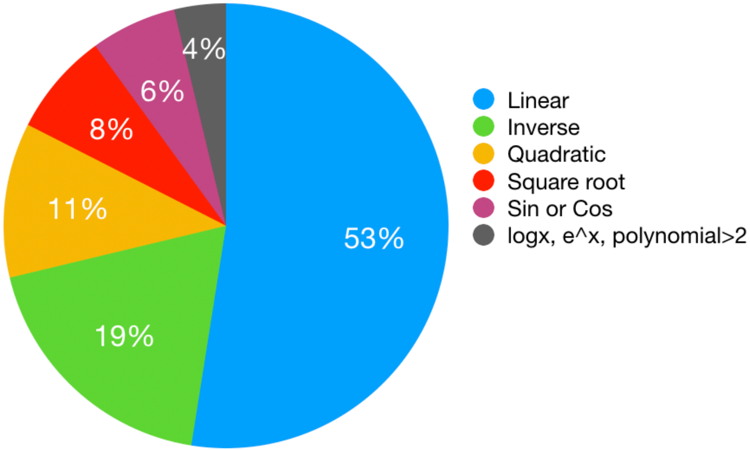
displace-ment[1]	$x(t_2) - x(t_1)$	$= \int_{t_1}^{t_2} dx$	$= \int_{t_1}^{t_2} \frac{dx}{dt}\, dt$	$= \int_{t_1}^{t_2} v(t)\, dt$
$\Delta\mathbf{v}$[1,2]	$v(t_2) - v(t_1)$	$= \int_{t_1}^{t_2} dv$	$= \int_{t_1}^{t_2} \frac{dv}{dt}\, dt$	$= \int_{t_1}^{t_2} a(t)\, dt$
impulse[3]	$p(t_2) - p(t_1)$	$= \int_{t_1}^{t_2} dp$	$= \int_{t_1}^{t_2} \frac{dp}{dt}\, dt$	$= \int_{t_1}^{t_2} F(t)\, dt$
work done *on* system[4]	$U(x_2) - U(x_1)$	$= \int_{x_1}^{x_2} dU$	$= \int_{x_1}^{x_2} \frac{dU}{dx}\, dx$	$= \int_{x_1}^{x_2} F(x)\, dx$

1.Kinematics 2.Newton's Laws Conservation of: 3.Momentum and 4.Energy

(White Brahmia 2023)

Infrastructure of physics models *(White Brahmia 2023)*

The Physics Hypertextbook (equations) 2023,
Glen Elert: https://physics.info/equations/



Conceptualizing function

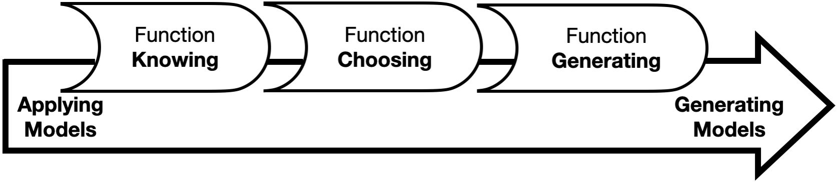
For every parent function, successful modeling relies on conceptualizing:

- graphical shape
- characterizing the rate of change as the independent variable gets bigger
- the behavior near 0 and at large values of the independent variable
- functional form first derivatives

Experts access this knowledge effortlessly, and use it to reason frequently and productively.

(Zimmerman et al., under review)

Expert reasoning with function



- first considered a model they knew from a similar context (e.g. circular motion invokes sinusoidal functions)
- if that failed, chose from the parent functions using their conceptual knowledge of the function's behavior
- as a last resort, generated an unfamiliar graph, considering
  - the neighborhood of physically significant points (1st derivative)
  - the behavior between those points (2nd derivative).

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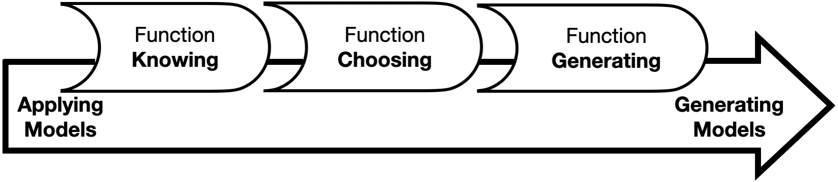

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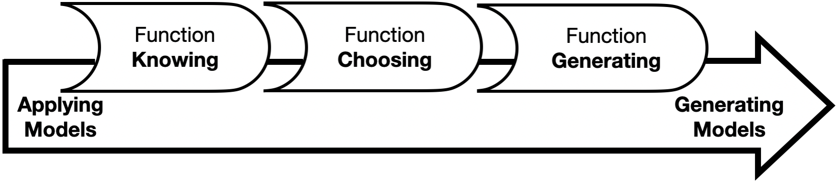

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Connecting to the physical world

Generating functions

- there may not be a simple analytical model to choose, especially in courses beyond the first year
- making approximations is a standard device for rendering a problem, or data set, more tractable
- we hope for a simple function, but often settle for the first few terms of its series expansion when modeling - and constrain the model to small values of the independent variable
- knowing how common approximations are used (especially Taylor series) and why, in a physics context, would help students of physics

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Calculus for students of physics

Students

- perceive math in physics as different from math in math
- struggle conceptualizing the mathematics used in physics, and it carries over to subsequent course-taking
- would like to understand when and why certain mathematics makes sense

Physics experts

- rarely break from reasoning, and making mathematical sense, with the sign, units, and properties of physical quantities
- develop models with many quantities that are related through the FTC
- develop models from a small number of common functions, and use series representations frequently to connect analytical solutions to the real world messiness
- discard valid mathematical models that don't predict what is observed in nature

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Towards a Transformative Calculus

Calculus can help by using quantity – including sign and units – from the natural sciences, the physical sciences, engineering and economics to help students develop a *conceptual* understanding of:

- rate and accumulation as quantity
- graphical features of common functions and their series representations
- first and second derivative reasoning when modeling covariational relationships
- how and why approximation methods are used

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Promising materials for calculus instruction

Developing a conceptual foundation

- Physics Invention Tasks (White Brahmia, Kanim, Boudreaux): students engage in quantification, inventing ratio or product quantities, rules or equations to characterize a variety of physical systems.

- Precaclulus: Pathways to Calculus (Carlson, Oehrtman, Moore, O'Bryan): Facilitates student construction of calculus ideas, including constant rate of change and linear function, changing rates of change, using covariation, vector quantities, sequence and series representation as approximation

Promising materials for calculus instruction

Calculus activities

- DIRACC Calculus(Thompson, Ashbrook, Milner) reasoning about quantities and relationships among quantities, FTC as relating rates of change and accumulations. Dynamic graphs help prime students for the ubiquitous reference to "goes like" reasoning their instructors use from the very first day *(Zimmerman, Olsho, White Brahmia, Boudreaux, Smith, Eaton, 2020)*.

- ACRA Framework: contact Jason Samuels directly for materials (jsamuelsbmcc@gmail.com)

- CLEAR Calculus (Oehrtman, Tallman, Reed, Martin) generalizes across contexts to extract common mathematical structure; rates of change, related rates, optimization, FTC, approximations, Taylor series+more.

Dugnad

"Dugnad" (i.e. barnraising) is jointly performed and unpaid and voluntary work of importance to the community or an individual. Dugnaders are usually carried out in a local community, as neighborly help in different situations, but sometimes also on a regional or national level.

Let's go, Dugnaders!!

Thank you!!